Applied Master's Thesis

Asset Bubble Detection in the Cryptocurrency Market

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I. Introduction

In the world of digital assets, it is not uncommon to see the price of cryptocurrencies rising by more than 500% in less than a year. For example, the price of Bitcoin increased from around 10,000$ at the start of September 2020 to around 64,000$ mid-April 2021, marking an enormous 530% increase in price. Similarly, cryptocurrencies have also become infamous due to the large crashes in price that often ensue drastic price increases. Recent fears of sustained elevated inflation levels and aggressive interest rate raises by central banks world-wide have caused major selloffs in financial markets. Digital assets appear to be one of the hardest hit markets with the total cryptocurrency market cap dropping more than 70% since mid-November 2021. Many financial experts have referred to this cryptocurrency sell-off as another “crypto bubble popping” and have drawn similarities to the pop of the dotcom bubble in 2001. Many investors and financial analysts have been searching for ways to get signals when a bubble is forming as well as signals when a bubble is about to burst.

Rational bubbles can be defined as bubbles that occur when asset prices rise rapidly due to investor’s belief that they will be able to sell the overvalued asset for greater returns later (Flood and Hodrick, 1990). As asset become riskier to hold the more overvalued it becomes and therefore investors expect higher returns. This continued higher profit seeking behaviour leads to even further asset price inflation and ultimately leads to the asset bubble bursting. There are two main types of rational bubbles that can occur: extrinsic and intrinsic bubbles. Intrinsic rational bubbles occur when investors systematically misestimate the fundamental value of an asset (Dale et al., 2005). These types of bubbles often occur in new technologies as there is no consensus on how these technologies should be valued fundamentally. Extrinsic rational bubbles occur when investors are faced with elevated uncertainty about the macroeconomic atmosphere. Therefore, extrinsic rational bubbles often occur due to investors relying too heavily on misinformation (Kyriazis et al., 2020).

There is currently no consensus in academic literature on how to best measure asset bubbles as rational bubbles can take the form of many different time trends such as a first order auto regressive process as well as many other more complex stochastic processes. Overall, there appear to be 3 frequently used models to define bubbles. The first and most traditional model, namely the asset-pricing model, considers a bubble to exist if the nominal value of an asset is higher than the fundamental value of an asset. In this case the fundamental value is the present value of the payoffs from the assets after taking into consideration all relevant information (Taipalus, 2012). Foster and Wild (1999) provide a second approach to modelling an asset bubble using a sigmoid or logistic curve. This methodology is useful when attempting to distinguish between the 3 main phases of bubble formation: expansion, inflexion, and saturation, but lacks the ability to measure multiple bubbles over a given period. The third most used approach to detecting both single and multiple bubbles is the Markov-switching Augmented Dickey-Fuller unit root test (MSADF) and was proposed by Hall et al. (1999).

This research section is structured as follows. Section II. provides and overview of the data used in this research. Section III. includes the two methodologies used for detecting bubbles: Log-Periodic Power Law Singularity (LPPLS) and the Generalised Supremum Augmented Dickey Fuller (GSADF). Section IV. interprets the results of the LPPLS confidence indicators and the GSADF test statistics. Section V. summarises and concludes.

II. Data

The data used for this research are the daily close prices of Bitcoin and Ethereum in USD from 1st January 2019 to the 30th June 2022 (downloaded from Yahoo Finance). The specified period of analysis for this research complements previous literature which predominantly focuses on detecting cryptocurrency bubbles prior to the year 2020. Bitcoin and Ethereum were chosen as proxies for cryptocurrency market as together they comprise more than 75% of the current total cryptocurrency market cap. The dataset contains no gaps as cryptocurrencies are traded day-round and year-round.

Figure 4: Evolution of Bitcoin and Ethereum log daily close prices from 1/1/2019– 30/6/2022

Note: This figure shows the log price evolution of Bitcoin and Ethereum from 1st January 2019 to 30th June 2022. The log daily closing price is used to scale relative price increases to their subsequent periods in time and graphically compare price movement differences between Bitcoin and Ethereum.

Figure 4 shows the log price evolution of both Bitcoin and Ethereum. The price evolution of Bitcoin and Ethereum are very similar over this period. A correlation coefficient of 0.927 further suggests that the largest market cap cryptocurrencies are highly correlated. This finding gives further credibility to using Bitcoin and Ethereum for further inferences on the cryptocurrency market. Throughout the 3.5-year period there appear to be several instances of explosive positive price behaviour that are followed by subsequent radical drops in price. Graphically there appears to be evidence of both smaller bubbles forming and bursting as well as potentially a large bubble that formed between March 2020 and August 2022. Summary statistics for the Bitcoin and Ethereum data used can be found in Table 1 below.

Table 1: Summary Statistics for the Log Daily Close Prices of Bitcoin and Ethereum

Variable	Obs.	Mean	SD	Min	Max	Skewness	Kurtosis
Log BTC	1272	9.547	0.374	8.082	11.121	0.362	-1.232
Log ETH	1272	6.335	0.516	4.434	8.479	0.353	-1.357

Note: This table shows the summary statistics for the log Bitcoin and Ethereum daily close price sample of from 1st January 2019 to 30th June 2022.

III. Methodology

This research uses the Log-Periodic Power Law Singularity (LPPLS) and the Generalised Supremum Augmented Dickey Fuller (GSADF) methodologies to detect bubbles in cryptocurrencies as they are the most employed methodologies in recent literature (Kyriazis et al., 2020). There are many different variations of both the LPPLS (Shu and Zhu, 2020; Xiong et al. 2019; Bianchetti et al., 2018; Cheah and Fry 2015; MacDonell 2014) and the GSADF (Corbet et al., 2018; Cheung et al., 2015; Philips et al., 2012) methodologies with each of them offering slight improvements.

LPPLS Model

The LPPLS model was first used by Johansen, Ledoit, and Sornette (JLS) in 2000 to help analyse and predict stock market bubbles. The LPPLS model is therefore also sometimes referred to in literature as the JLS model. In this model, a bubble is defined as an asset price increase that occurs at a rate that is higher than an exponential increase. Therefore, in a bubble regime the asset price "decouples from its intrinsic fundamental value and displays two typical characteristics: the transient super-exponential growth and the accelerating log-periodic volatility fluctuations" (Shu and Zhu, 2020). In this case a bubble is a competing process of both a positive feedback loop of higher anticipated returns for traders and investors and a negative feedback spiral of a potential asset price crash. According to Yan (2011) the LPPLS model is based on the synthesis of three areas of knowledge: 1. Economic theory of rational expectation bubbles, 2. Behavioural finance of imitation and herding of traders, 3. Mathematical and statistical physics of bifurcations and phase transitions. The exact model used for this research can be found in equation (1) below and is taken directly from the original LPPLS model by Johansen et al. (2000).

$$E\,[\ln p(t)] = \ln(p_c) + B_0(t_c - t)^{\beta} + B_1(t_c - t)^{\beta}\cos\left(\omega \ln(t_c - t) - \phi\right) \quad (1)$$

- $E\,[\ln p(t)]$: expected log price of the asset when the bubble ends
- $\ln(p_c)$: log price of asset at the critical time c when bubble has not ended
- β: measure of super exponential growth
- B_0: power law acceleration amplitude
- B_1: log-periodic oscillations amplitude
- t_c: most probable time of crash
- ω: scaling ratio of the temporal hierarchy of oscillations
- ϕ: time scale of the oscillations

The model consists of 3 main components that represent the development of an asset bubble. First, $\ln(p_c) + B_0(t_c - t)^{\beta}$ represents the power law, which tends to infinity when $\beta < 1$ and the most probable time of a crash t_c is reached. Second, $B_1(t_c - t)^\beta$ represents the amplitude of the oscillations in the asset price and drops to zero at t_c. Third, cos $(\omega \ln(t_c - t) - \phi)$ is a measure of the frequency of oscillations which tends towards infinity at t_c. This model is used to calculate the LPPLS confidence indicator and is applied to the Bitcoin and Ethereum log daily close price data using the *lppls* package in Python (Nielsen, 2022). A LPPLS confidence indicator value greater than 0 indicates a state of exuberance, with higher values indicating a higher degree of exuberance. The full code used for implementing the LPPLS model can be found in section *VII. Appendix*.

GSADF Test

The GSADF test is an extension of the simple Augmented Dickey Fuller (ADF) test and was first introduced by Phillips et al. (2012) to detect bubbles in the housing market. To understand the advantages of the GSADF test over a simple ADF test one must first understand how an ADF test is calculated. Even more complex ADF methodologies all start with a simple ADF regression equation that can be seen in equation (2).

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-1} + \epsilon_t \qquad (2)$$

ΔY_t is a generic time series, ΔY_{t-1} with i = 1, …, m are lagged first differences of the series to account for serial correlation and, and β_1, β_2, δ, and α_i with i = 1, …, m are the coefficients. The null hypothesis is that a unit root exists (H_0: $\beta_2 = 0$), the alternative hypothesis that the time series is exuberant over the entire sample is (H_1: $\beta_2 > 0$). Using a simple ADF test to detect asset bubbles is problematic when an asset bubble only forms over a sample of time in the time series and the bubble formation does not cover the entire time series duration. Evans (1991) further highlighted this problem by showing that the simple ADF test frequently led to findings of spurious stationarity in time series that included periodically collapsing bubbles and that contained multiple periods of exuberance.

Phillips et al. (2011) were the first to propose an ADF test that was able to detect a single bubble formation and collapse over only a subsample of the entire time series. This test is called the Supremum Augmented Dickey Fuller (SADF) test and is recursively estimated to allow for an asset bubble to only exist for a short period throughout the time series. The null hypothesis of the SADF is the same as that of a simple ADF test, however, the alternative hypothesis of the SADF test is that of explosive asset price behaviour in only some part of the sample. Finally, Phillips et al. (2012) proposed the Generalised Supremum Augmented Dickey Fuller (GSADF) which has the same alternative hypothesis as the SADF test, however, it allows for a greater number of subsamples to exhibit explosive behaviour. The GSADF is therefore better suited to detect multiple bubble formations and crashes over a time series compared to the simple ADF test and the SADF test. The GSADF test is applied to the daily log close prices of both Bitcoin and Ethereum by using the R package *exuber* (Vasilopoulos et al., 2021). The full code for implementing the GSADF model can be found in section *VII. Appendix*.

IV. Results & Discussion

Both Bitcoin and Ethereum have experienced several price crashes between 1st January 2019 and the 30th June 2022. This research defines a crash in either Bitcoin or Ethereum as a 25% or larger drop in price after creating a local or global maximum. As Bitcoin and Ethereum are volatile assets, 25% is chosen as the threshold for a crash to avoid any daily or even interday noise. Table 2 and Table 3 below show all the time periods between 1st January 2019 and 30th June 2022 where the price of Bitcoin and Ethereum crashed 25% or more respectively. Both Bitcoin and Ethereum experienced 6 significant crashes throughout this time period with the most significant crash for both occurring in Q1 and Q2 of 2022.

Table 2: Bitcoin Crashes between 1/1/2019 and 30/6/2022

Number	Peak Day	Peak Price ($)	Crash End Day	Crash End Price ($)	Crash size
1	26/06/2019	13016.23	30/07/2019	9607.42	-26%
2	05/08/2019	11805.65	25/11/2019	7146.13	-39%
3	13/02/2020	10214.38	12/03/2020	4970.79	-51%
4	08/05/2021	58803.78	20/07/2021	29807.35	-49%
5	08/11/2021	67566.83	22/01/2022	35030.25	-48%
6	30/03/2022	47465.73	18/06/2022	19017.64	-60%

Note: The Bitcoin crashes listed above are all 25+% in magnitude and occurred between 1/1/2019 and 30/6/2020 All data points were taken from Yahoo Finance.

Table 3: Ethereum Crashes between 1/1/2019 and 30/6/2022

Number	Peak Day	Peak Price ($)	Crash End Day	Crash End Price ($)	Crash size
1	05/01/2019	155.64	06/02/2019	104.92	-33%
2	26/06/2019	336.75	29/08/2019	169.52	-50%
3	19/09/2019	221.28	24/09/2019	168.11	-25%
4	15/02/2020	264.73	13/03/2020	133.20	-50%
5	10/11/2021	4636.17	22/01/2022	2405.18	-48%
6	05/05/2022	2749.21	18/06/2022	993.64	-64%

Note: The Ethereum crashes listed above are all 25+% in magnitude and occurred between 1/1/2019 and 30/6/2020 All data points were taken from Yahoo Finance.

Using the *lppls* package in Python, I generate the LPPLS confidence indicators for both Bitcoin and Ethereum log daily close price data from 1st January 2019 until 30th June 2022. Even though, the LPPLS confidence indicators are also able to detect negative bubble formation, I limit my analysis to positive bubble formation. Positive bubbles have one of two outcomes once the highest point of exuberance is reached. Either a volatile sideways plateau emerges, or there is a crash. Figure 4 and Figure 5 show the LPPLS confidence indicators for both Bitcoin and Ethereum over the specified period respectively.

Figure 5: LPPLS Confidence Indicator for Bitcoin from 1/1/2019 until 30/6/2022

Note: This graph was created using the lppls Python package and using Bitcoin log daily close data from 1/1/2019 until 30/6/2022. The orange line represents the LPPLS confidence indicator, and the black line is the log daily close price of Bitcoin.

The LPPLS confidence indicator values range from 0 to 1, with 0 indicating no state of exuberance and 1 indicating a very strong state of exuberance in the fitting window. LPPLS confidence indicator values that are close to 0 need to be carefully analysed as they might only exist due to over-fitting. Therefore, this research only takes into consideration LPPLS confidence indicators that have a value greater than 0.1. According to the clusters of LPPLS confidence indicator values in Figure 5 that are greater than 0, there are 5 bubbles that formed in Bitcoin during this time period. The 5 bubbles were formed in Q1-Q2 of 2019, Q1 of 2020, Q2-Q3 of 2020, Q4 of 2020 - Q1 of 2021, and Q3-Q4 of 2021, according the LPPLS confidence indicator values are. The only Bitcoin bubble that the LPPLS does not appear to capture is the bubble that popped on the 30/3/2022. There are 3 potential reasons as to why this bubble was not detected. First, this bubble could be seen as an extended crash that started in Q4 2020. Second, the window sizes of the LPPLS model are misspecified and therefore the LPPLS confidence indicator does not meet the threshold of 0.1 to be considered a reliable indicator of bubble formation. Third, the cut-off level of 0.1 for an LPPLS confidence indicator to act as a signal of exuberance is set too high.

Figure 6: LPPLS Confidence Indicator for Ethereum from 1/1/2019 until 30/6/2022

Note: The graph was created using the lppls Python package and using Ethereum log daily close data from 1/1/2019 until 30/6/2022. The orange line represents the LPPLS confidence indicator, and the black line is the log daily close price of Ethereum.

The clusters in Figure 6 indicate 4 Ethereum bubble formations that took place between 1st January 2019 and 30th June 2022. The 4 bubbles were formed in Q2 2019, Q1 2020, Q2 -Q3 of 2020, and Q4 of 2020 – Q2 2021, according the LPPLS confidence indicator values. The LPPLS confidence indicator is not able to detect the bubble formation that occurred in Q3-Q4 2021 and Q1 2022. The potential reasons as to why these two bubble formations were undetected are the same as those described for Bitcoin.

The LPPLS confidence indicator appears to be a valuable tool, however, it is not 100% accurate and should not be relied upon exclusively by investors to detect bubbles in the cryptocurrency market. In addition, the LPPLS can be complicated to fit as there are 7 parameters which are prone to misspecification errors. There are two potential improvements to this model that are also discussed by Shu and Zhu (2020). The first improvement is to use hourly price data instead of daily data. This helps to improve the accuracy of when the investor is warned. The second improvement is using the CMA-ES rates algorithm to calculate the three non-linear parameters t_c, β, and ω, which provides some accuracy improvements (not possible to implement in this research study as it requires parallel computing).

Figure 7: Bitcoin GSADF test statistic from 1/1/2019 until 30/6/2022

Note: This graph was created using the package exuber in R with Bitcoin log daily price close data from 1/1/2019 – 30/6/2022. The red line signifies the critical value at which the null hypothesis is rejected. The grey shaded regions are areas when the price is in a bubble as the GSADF test statistic is greater than the critical value.

Figure 7 shows the Bitcoin GSADF test-statistic that was generated recursively over several time windows from 1st January 2019 until the 30th June 2022. The red dotted line represents the critical value above which the null hypothesis of a unit root is rejected, and the alternative hypothesis of exuberance is accepted. Figure 6 displays 7 periods of exuberance for Bitcoin over the specified period. Using the datestamping function in the R package *exuber* I create the exact dates of exuberance that are seen in Figure 6. These exact dates can be found in Table 4 below.

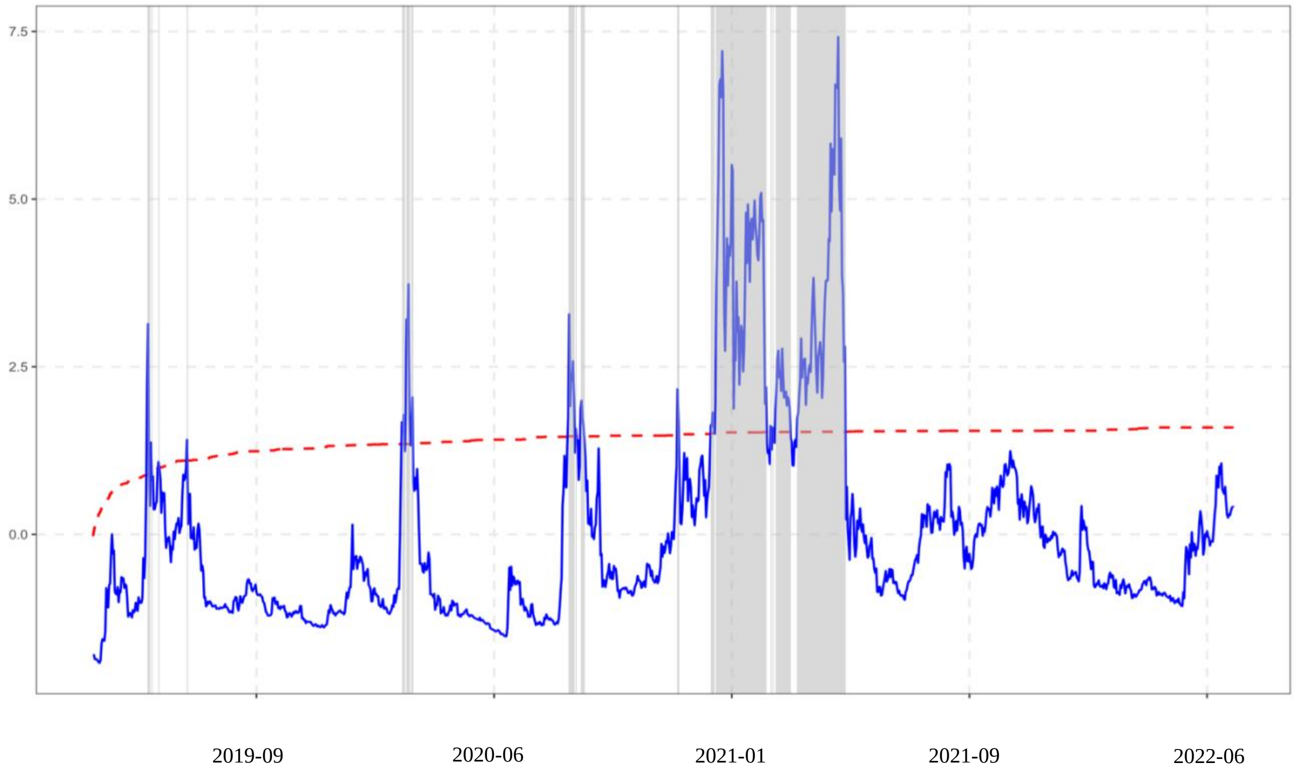
Table 4: Datestamping Bitcoin Bubbles using the GSADF test

Number	Bubble Start Date	Bubble End Date
1	01/04/2019	10/04/2019
2	08/05/2019	02/06/2019
3	16/06/2019	29/06/2019
4	04/11/2020	30/11/2020
5	15/12/2020	26/01/2021
6	27/01/2021	20/04/2021
7	29/04/2021	01/05/2021

Note: The dates in this table were created using the datestamping function in the R package exuber after having calculated the GSADF test statistics. The data used is daily Bitcoin log price from 1st January 2019 until 30th June 2022.

The GSADF test is only able to warn investors about 3 of the 6 major bubble formations that occurred from the 1st January 2019 until the 30th June 2022. It appears that the GSADF test only rejects the null hypothesis in extreme cases of exuberance as seen in Q2 of 2019, Q4 of 2020, and Q1 of 2021. Due to its lack of sensitivity it fails to detect bubble formations that occur over more extended periods and do not form more slowly over several weeks.

Figure 8: Ethereum GSADF test statistic from 1/1/2019 until 30/6/2022



Note: This graph was created using the package exuber in R with Ethereum log daily price close data from 1/1/2019 – 30/6/2022. The red line signifies the critical value at which the null hypothesis is rejected. The grey shaded regions are areas when the price is in a bubble as the GSADF test statistic is greater than the critical value.

Table 5: Datestamping Ethereum Bubbles using the GSADF test

Number	Bubble Start Date	Bubble End Date
1	14/05/2019	26/05/2019
2	25/06/2019	26/06/2019
3	06/02/2020	18/02/2020
4	30/07/2020	16/08/2020
5	22/11/2020	24/11/2020
6	27/12/2020	18/05/2021

Note: The dates in this table were created using the datestamping function in the R package exuber after having calculated the GSADF test statistics. The data used is daily Ethereum log price from 1st January 2019 until 30th June 2022.

Similarly, to the Bitcoin GSADF test, the Ethereum GSADF test is only able to warn investors about 3 out of the 6 major crashes that occurred between 1st January 2019 and 30th June 2022. This can be seen by the datestamps in Table 5 and Figure 8 only giving warnings of price exuberance before 3 of the 6 major crashes. A possible improvement to the GSADF test is changing the window and step sizes before running the recursive estimations and seeing which window and step sizes yield the highest accuracy of bubble formation detection. It appears that both the LPPLS model and the GSADF test have strengths and weaknesses regarding the detection of cryptocurrency bubbles. On the one hand, the LPPLS is more sensitive and has a higher likelihood of providing signals when bubbles are forming as compared to the GSADF test. On the other hand, the GSADF is less noisy and provides more continuous signals of bubble formation compared to the LPPLS model.

V. Conclusion

In this research I aimed to identify whether it is possible to detect bubble formation in Bitcoin and Ethereum. First, I provide explanations as to why bubble detection in cryptocurrencies is important and what methodologies have been used to date. Second, I explain how I use the Log-Periodic Power Law Singularity (LPPLS) and the Generalised Supremum Augmented Dickey Fuller (GSADF) methodologies to detect bubbles in both Bitcoin and Ethereum. Third, I use both R and Python to calculate both the LPPLS confidence indicators and the GSADF test statistics. Lastly, I interpret the results and give reasoning for the findings.

Bitcoin and Ethereum both experienced multiple boom and bust cycles from 1st January 2019 until the 30th June 2022. In total, 6 price crashes of 25% or more were recorded for both of the high marketcap cryptocurrencies. I find that overall, the LPPLS methodology is able to detect more bubble formations as compared to the GSADF test. The LPPLS methodology is more sensitive and therefore also produces more noisy signals as compared to the GSADF test. To use either model as an accurate bubble detection tool, further research needs to be done on how to determine the optimal window and step sizes for each model. In addition, further research should explore on-chain methodologies and assess their validity as fundamentals for cryptocurrencies.

VI. References

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VII. Appendix

VII.1. Python Code for LPPLS methodology

```
#Imports
from lppls import lppls, data_loader
from lppls import lppls_cmaes
import pandas as pd
import numpy as np
import math
from datetime import datetime as dt
%matplotlib inline
import mplfinance as mpf
import matplotlib.pyplot as plt

#Import daily price data
df = pd.read_csv('MSFE Thesis Data.csv')

#Cleaning the data and turning prices into log prices
df['ln_BTC']=np.log(df['BTC'])
df['ln_ETH']=np.log(df['ETH'])

#Create time and price for observation data
time = [pd.Timestamp.toordinal(dt.strptime(t1, '%d/%m/%Y')) for t1 in df['Date']]
price_BTC = df['ln_BTC'].values
price_ETH = df['ln_ETH'].values

# Create observations array (this is what the LPPLS model expects)
obs_BTC = np.array([time, price_BTC])
obs_ETH = np.array([time, price_ETH])

# Max number of searches is 25 (suggested from literature)
max_searches = 25

# Create the model
lppls_model_BTC = lppls.LPPLS(observations=obs_BTC)
lppls_model_ETH = lppls.LPPLS(observations=obs_ETH)

# Fit the model to the data
tc, m, w, a, b, c, c1, c2, O, D = lppls_model_BTC.fit(max_searches)
lppls_model_BTC.plot_fit()
tc, m, w, a, b, c, c1, c2, O, D = lppls_model_ETH.fit(max_searches)
lppls_model_ETH.plot_fit()

# Confidence indicator calculation
res_BTC = lppls_model_BTC.mp_compute_nested_fits(
    workers=8,
    window_size=120,
    smallest_window_size=5,
    outer_increment=1,
```

```
    inner_increment=5,
    max_searches=25,
)

lppls_model_BTC.plot_confidence_indicators(res_BTC)

res_ETH = lppls_model_ETH.mp_compute_nested_fits(
    workers=8,
    window_size=120,
    smallest_window_size=5,
    outer_increment=1,
    inner_increment=5,
    max_searches=25,
)

lppls_model_ETH.plot_confidence_indicators(res_ETH)

# Store indicators
res_BTC_df = lppls_model_BTC.compute_indicators(res_BTC)
res_ETH_df = lppls_model_ETH.compute_indicators(res_ETH)

# Store indicators
for i in range(0,(len(res_BTC_df))):
    if res_BTC_df.loc[i,'pos_conf']==0:
        res_BTC_df = res_BTC_df.drop([i])

for i in range(0,(len(res_ETH_df))):
    if res_ETH_df.loc[i,'pos_conf']==0:
        res_ETH_df = res_ETH_df.drop([i])

# Resetting the index for BTC and ETH dataframes
res_BTC_df = res_BTC_df.reset_index()
res_ETH_df = res_ETH_df.reset_index()

# Changing the time to non-ordinal
for i in range(0,(len(res_BTC_df))):
    l = dt.fromordinal(int(res_BTC_df.loc[i,'time']))
    res_BTC_df.loc[i,'time'] = l.strftime('%d/%m/%Y')

for i in range(0,(len(res_ETH_df))):
    l = dt.fromordinal(int(res_ETH_df.loc[i,'time']))
    res_ETH_df.loc[i,'time'] = l.strftime('%d/%m/%Y')

# Exporting dataframes as csv
res_ETH_df.to_csv('ETH_bubble_dates.csv', index=False)
res_BTC_df.to_csv('BTC_bubble_dates.csv', index=False)
```

VII.2. R Code for GSDAP methodology

```
# Install packages
install.packages("exuber")
library(exuber)
install.packages("exuberdata")
library(exuberdata)
install.packages("ggplot2")
library(ggplot2)
install.packages("zoo")
library(zoo)
install.packages("xts")
library(xts)

# Plotting the price of both Bitcoin and Ethereum
df_1 <- read.table("/Users/ChristopherJack1/Desktop/Master's Thesis/MSFE Thesis Data for R.csv",
header=TRUE, sep=",", dec=".")
df_1

# Create dataframes for BTC and ETH seperately
BTC = df_1["BTC"]
ETH = df_1["ETH"]

# Apply radf to both
radf_BTC <- radf(BTC)
radf_ETH <- radf(ETH)

# Using exuber package to create test-statistics
summary(radf_BTC)
summary(radf_ETH)

# Plotting the tests
plot_BTC_gsadf <- autoplot(radf_BTC)
plot_BTC_gsadf

plot_ETH_gsadf <- autoplot(radf_ETH)
plot_ETH_gsadf

# Determining exact dates of the bubbles
date_stamp_BTC = datestamp(radf_BTC)
date_stamp_ETH = datestamp(radf_ETH)

# Plotting datestamp results
autoplot(date_stamp_BTC)
autoplot(date_stamp_ETH)
```